

## **The Schur complement method and solution of large-scale geophysical problems**

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The Schur complement domain decomposition method is implemented into a linear solver to achieve high performance on parallel computers with distributed memory. One of the main advantages of the method is its robustness and its scalability, even for cases with strongly varying matrix coefficients. Two types of parallelism can be exploited, namely, solving independent subdomains in parallel and using multiple processors per subdomain. Each subdomain is treated, coupled with the interface subdomain, in order to find the Schur complement matrix. In addition, as each subdomain is processed independently, the optimization of the memory usage is facilitated.

We present the algorithm and the program which can be used in many geophysical simulations where more system demanding computations are required.

## **A numerical investigation of the influence of density anomalies in the lower mantle on the geoid**

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The influence of the two near-equatorial, antipodal Large Low Shear Velocity Provinces (LLSVPs) in the lower mantle on global mantle dynamics is a topic of major interest in geodynamics. It was found in seismic studies that LLSVPs exhibit excess density with respect to the surrounding mantle which means that they are not thermal superplumes, as previously thought. This has important implications for the overall convection style of the Earth's interior. It also changes the interpretation of the correlation between LLSVPs and observed positive geoid anomalies. If the anomalies were hot superplumes, they would drive a rising flow in the mantle and thus cause positive geoid anomalies due to dynamic topography of the surface. Yet, since the anomalies were found to exhibit excess density, such flow is expected to be much less significant and the associated geoid anomalies would be smaller than for superplumes. The excess density, in turn, could then be the major cause of the positive geoid anomalies. Even though density anomalies in the lower mantle are in general expected to have a relatively small influence on the geoid due to their great distance from the surface, large volumes with wide lateral extent, as is the case for LLSVPs, could still produce a strong geoid signal and be responsible for the observed positive geoid anomalies. Since both density excess of the anomalies and dynamic effects (resulting in dynamic topography) have an influence on the geoid signal, we investigate both effects on the geoid in fully dynamic two-dimensional mantle convection models with cartesian and spherical axisymmetric geometries.

# Self-organized layer formation in a solidified magma ocean

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At an early stage of the Earth's and other planets' thermal evolution, the energy due to accretion or possibly due to subsequent giant impacts should have been sufficient to cause substantial melting of the mantle, creating a magma ocean (MO) [Solomatov (2007)]. Solidification of a thermally well-mixed MO is expected to proceed from the bottom upward, because the solidus temperature increases with depth more rapidly than the adiabat. During crystallization of the MO, the residual fluid is progressively enriched in iron and incompatible elements, resulting in a gravitationally unstable density stratification [e.g. Elkins-Tanton *et al.* (2005)].

After solidification of the MO, cooling of the planets is largely controlled by convective processes. These do not necessarily lead to a homogenization of the planet's interior, but can be self-organized into chemically distinct reservoirs. We perform numerical modeling of thermochemical convection in a 2D Cartesian geometry to investigate the self-organized formation of convecting layers in an initially unstably stratified fluid that is heated from below and cooled from above. Different initial mantle temperatures as well as a temperature-dependent rheology are taken into account.

We show that after an initial overturn, compositionally distinct layers evolve that exist over geologically relevant timescales. Formation and breakage of layers substantially influence the surface heat flow. Breakage of a layer, for example, leads to a jump in the heat flow. Different initial mantle temperatures affect the onset of convection at the surface which could be interpreted as the onset of plate tectonics. In an initially cold mantle, the onset of plate tectonics occurs later than in a mantle that is moderately heated or hot.

If a temperature-dependent rheology is applied, the unstable stratigraphy is not completely inversed during the overturn. Due to the high viscosity of the surface layer, the late-stage crystallized materials with the highest intrinsic densities remain at the surface. This may have a strong impact on the heat budget and convective style of the planet, as heat-producing elements are not only enriched at the core-mantle boundary but as well at the surface.

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# Numerical convection modelling of a compositionally stratified lunar mantle

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Convection modelling of the lunar interior is generally done in simplified models with either a homogeneous composition or only one compositionally distinct layer, depending on the focus of the study (e.g. [1, 2, 3, 4]). When such a compositionally distinct and relatively dense layer is used, the focus is typically on the overturn of the lunar mantle, due to the gravitational instability which originated from the crystallisation of an early lunar magma ocean.

It is generally assumed that the Moon accreted as a hot planetary body (independent of which process led to Moon formation). The Moon then consisted of a global magma ocean, which crystallised upon cooling. Calculations on this crystallisation process show that the result was a layered mantle, covered by a plagioclase flotation crust. The last material to crystallise at shallow depth below this crust was a layer rich in high density ilmenite ( $\text{FeTiO}_3$ ) [5]. The high density ilmenite-rich layer at shallow depth was gravitationally unstable and this likely resulted in an overturn of the lunar mantle.

The dense ilmenite-rich layer, which crystallised at shallow depth beneath the crust, has been included when modelling the formation of the ilmenite-rich basalts found at the lunar surface [6] or to study the possible formation of an ilmenite-rich core in the lunar interior [3]. However, the compositional layering in the mantle below the ilmenite-rich layer is usually neglected and a constant background composition is used in the modelling instead. The deeper layering is likely to at least influence the timing of the overturn, but also the general dynamics. Therefore, this study investigates the influence of a more realistic mantle stratification on the overturn of the lunar mantle, using multi-component thermo-chemical convection models.

Thermo-chemical convection models were performed, using a 360 degree cylindrical finite element mesh. The convection equations for an incompressible, infinite Prandtl number fluid were solved using an extended Boussinesq approximation, which includes both viscous dissipation and adiabatic heating. Composition is described using tracer particles, advected by the flow. The initial setup consist of compositional layering as originates from a crystallising magma ocean. The density and the number of layers is varied to study the influence on both thermal and chemical mantle evolution.

These models show that the more detailed initial layering influences the timing and dynamics of lunar mantle overturn and associated basalt production. A clinopyroxene, pigeonite and olivine layer below the ilmenite-rich layer, with a density which is slightly higher than the densities of the mantle layers below, results in a significantly earlier overturn compared to a model containing only the ilmenite-rich layer.

## Acknowledgements

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# Lateral core mantle boundary heat flux variations as a model of Martian paleomagnetic field

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## Abstract

The presence of a strong crustal magnetization on Mars [1] indicates an ancient magnetic field. The dichotomy in the crustal magnetization between the northern and southern hemisphere supports the idea of a hemispherical magnetic field at the late stage of the dynamo action. The flow in the underlying core is strongly influenced by the mantle convection. A significant feature of the Martian mantle dynamics is the low degree convection [2] leading to a lateral inhomogeneous heat flux at the Core Mantle Boundary (CMB). This affects the dynamics of the liquid iron core. We investigate the possible effect of the CMB heat flux pattern on the core dynamo action using numerical simulations. The Martian core is modelled as a rotating spherical shell of conducting fluid, where the flow is driven only by thermal convection since we assume no solid inner core.

The lateral heat flux variations at the CMB are considered as degree-1 heat flux perturbations. We systematically investigate the influence of the amplitude of the perturbation and its tilt angle with respect to the rotation axis on the magnetic field configuration. In the hemisphere of higher heat flux the vigor of the dynamo action is amplified while in the other hemisphere the convection is weakened.

A degree-1 CMB heat flux pattern oriented along the rotation axis establishes a strong equatorial asymmetric temperature profile. As a consequence an equatorial asymmetric convection mode is established, since the cooling is more efficient in the southern hemisphere. The action of the Coriolis force on advection along the latitudinal temperature gradient (meridional circulation) forces strong zonal winds, which are retrograde in the north and prograde in the south. Most of the kinetic energy is then carried by axisymmetric zonal flows. The dipole character of the magnetic field reduces due to weakening the columnar convection. Additionally, shearing in the boundary between the zonal wind cells, produces strong magnetic field which moves southwards, if the relative perturbation increases. The

flow is dominated by equatorial asymmetric, axisymmetric motion and hemispherical configuration of magnetic field is preferred (figure 1).

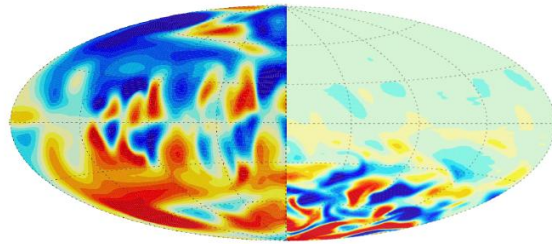


Figure 1: Radial magnetic field at the core mantle boundary for the reference case (constant flux) on the left half, and the strong perturbed system on the right. The dipolar morphology is lost, and a hemispherical dynamo is established.

## References

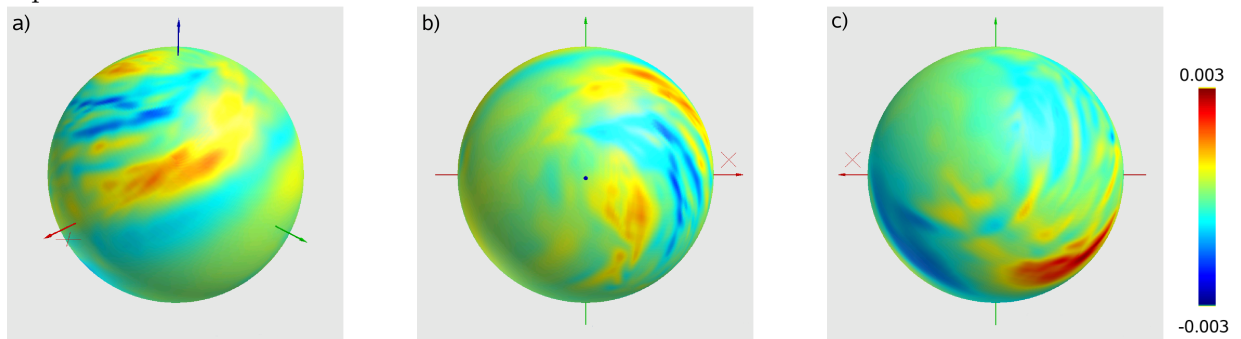
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# Precession driven dynamos

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It is widely accepted, that the planetary magnetic fields are powered by a magnetohydrodynamic dynamo-process. So far theoretical studies and numerical simulations have mostly assumed that the flow generating the dynamo-process is driven by buoyancy forces. But also precession can drive the dynamo, as first suggested by Bullard in 1949. A precession-driven laminar flow is mainly toroidal and cannot maintain a dynamo. However, experimental and numerical studies show that these basic flows are unstable and several kind of wave-like instabilities are generated. Therefore precession can also be regarded as a viable driving-mechanism of a core flow generating the planetary magnetic fields.

So far we have used a spherical, finite-volume code, already used for the simulation of convection-driven dynamos and mantle convection, to solve the equations of a precession-driven dynamo in a spherical shell. We investigated a full MHD-dynamo in a spherical shell, similar to that of Tilgner [1]. This flow can maintain a magnetic field but the magnetic field structure is not very similar to that of the Earth. For example the radial magnetic field at the outer boundary is not dipole dominated.



**Figure 1:** Snapshot of the radial magnetic field at the outer boundary from one side a), positive  $z$ -direction b) and from negative  $z$ -direction c). Parameters: precession rate  $\Omega = -0.3$ , obliquity  $\alpha = 60^\circ$ , Ekman number  $E = 3 \cdot 10^{-4}$ , magnetic Prandtl number  $Pm = 2$ , inner core size  $r_i = 7/13$ .

Furthermore the non-sphericity of the planetary bodies trigger some crucial instabilities. However, up to now the only available full MHD study of precessing spheroids is [2]. Their preliminary results for a full spheroid showed that this topographic coupling offers more favourable conditions for the generation of a sizeable dipole component of the magnetic field. We shall discuss how the ellipticity of the planets can be included in our calculations through the use of a non-orthogonal grid.

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# Sea floor flattening as response to plate deformations in mixed-mode heated self-consistent mantle convection



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There is a longstanding debate about the cause of the flattening observed in sea floor bathymetry at old ages. The two classic models, the half space cooling model (HSCM) and the plate model (PM), both describe the dependence of topography, heat flow, and geoid on the age of the plate [1]. They are based on two-dimensional heat conduction and adequately describe Earth's sea floor younger than 80 Myrs. At older ages the depth of the sea floor is shallower than predicted by HSCM [2]. Applying a self-consistent numerical model of mixed-mode heated mantle convection, we find that the flattening of the surface topography is dynamically plausible and that it is controlled by the deformations of the surface plate, which appear with increasing heating rates. Stein et al. [3] argued that mantle convection tends to change from a mobile-lid to a stagnant-lid type of convection with increasing internal heating rate. We observe a typical surface topography in each of the end-member cases and in between the topography shows characteristics of both regimes. In the transitional interval we find that the topography follows a sqrt-age-dependence at young ages and starts to oscillate around an equilibrated level when it becomes older. This behaviour is likewise observed for the boundary of the plate. While HSCM adequately explains topography at young ages, it underestimates the values at the older ages, which positively deviate. The PM perfectly follows the time averaged topography for all ages, while the oscillations can be understood as a time-dependent deviation.

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# Numerical models of salt dome formation by downbuilding: the role of sedimentation rate, viscosity contrast and other parameters

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## 1 Introduction

While the formation of salt domes as process of upbuilding has been studied extensively as an application of the classical Rayleigh-Taylor Instability (e.g. Ramberg, 1967), fully consistent fluid dynamical models of downbuilding salt diapirs (i.e. syndepositional diapirism) are rare (e.g. van Keken et al. 1993; Chemia et al, 2007). Here we systematically study the effect of the sedimentation rate  $v_s$ , the viscosity contrast  $m$  between salt and overburden, the wavelength  $\lambda$ , and the initial amplitude  $\delta$  of the perturbation on the success or failure of downbuilding and the geometrical shape of salt diapirs

## 2 Governing equations and model set up

The conservation equations of mass, momentum and composition are solved for a three component system, consisting of an initially flat salt layer with a viscosity ratio  $m$  times softer than the sediments, a denser sediment layer and a zero density much softer “sticky air” layer. The 2D Finite Difference code FDCON based on a stream function formulation is used in combination with a marker approach based on a predictor-corrector Runge-Kutta 4<sup>th</sup> order scheme.

Initially no sediments are present. To initiate downbuilding, sedimentation is modelled by successively elevating the initially perturbed level of the sediment layer by a prescribed sedimentation rate and transforming all “sticky air” particles below this level into sediments. Due to the small initial perturbation differential sediment loading drives the salt towards the centre of the future diapir. If the top of this diapir rises faster than the sedimentation level until the complete initial salt layer is drained downbuilding is defined to be successful. If sedimentation is too fast, the salt layer may only reach the pillow stage, then it is buried completely by the stiff sediment layer. This case defines failed downbuilding

## 3 Model results

Variation of the four model parameters have a strong effect on the resulting diapir shape. Small  $m$  and  $v_{sed}$  result in flat and broad domes, higher values leads to narrower and higher domes. Small  $\delta$  -values result in narrow stems and wide diapir heads, larger values lead to domes with subvertical side walls. The boundary between successful and failed downbuilding can be constrained within the 4-dimensional parameter space: Failing of

down-building occurs at high  $m$  or  $v_{sed}$  and at small  $\delta$ . Variation of  $\lambda$  shows that downbuilding is most successful within a  $\lambda$ -range around  $2\pi h_{salt}$  of about 1 order of magnitude width.

## 4 Discussion and conclusion

If  $v_{sed}$  is scaled as

$$v'_{sed} = v_{sed} \frac{\eta_{salt}}{h_{salt}^2 \Delta\rho g} \quad (1)$$

with  $\eta$  as viscosity,  $h$  as thickness,  $\Delta\rho$  as density difference and  $g$  as gravitational acceleration, then a simple channel flow model predicts that the critical sedimentation velocity separating successful from failed downbuilding is given by

$$v'_{sedcrit} = C_1 \frac{1}{2} \delta_0' \rho_0' \frac{k'^2}{k'^2 + C_2} \quad (2)$$

where the lengths are scaled by  $h_{salt}$  and the density is scaled by  $\Delta\rho$ . This model is confirmed to first order by the numerical experiments, which allows to determine the constant  $C_1$  and  $C_2$  as 0.026 and 0.09, respectively. However, non-linear effects due to laterally varying sediment amounts are observed and lead to moderate deviations from the simple relation (2).

We conclude that our numerical experiments and the resulting relation (2) allow to predict whether a salt layer will evolve into a salt dome by downbuilding during sedimentation, and, in reverse, may be used to constrain past sedimentation rates.

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Title: "Initial conditions, chaotic branching and feedback-stabilization in numerical dynamos"

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Abstract:

Due to its weak dipole field, Mercury exhibits a close magnetopause and is thus subject to strong magnetospheric fields caused by the magnetopause currents. At the surface, such fields have a magnitude comparable to that of the planetary internal field. It has been suggested that the behavior of planetary dynamos in the presence of external magnetic field sources may differ from the solutions found for isolated dynamos. Therefore, Mercury's dynamo is likely to be affected by the field at the core-mantle boundary (CMB) by the magnetospheric currents.

We present results from a numerical study, that shows how initial conditions and external field sources (i.e., a magnetospheric feedback) affect the stability of dynamo solutions. For the initialization of the magnetic field, we use a seed-field with toroidal and poloidal components. For weak convective forcing (slightly supercritical Rayleigh numbers) and moderate magnetic diffusivities (viscous to magnetic diffusivities ratio of  $Pm=2/3$ ), the dynamo reaches a saturation of magnetic field (a strong field solution) for strong seed-field values. However, it may also reach a weaker stable solution (a weak field solution) when weak seed-fields are used. In general, the magnetic energy of the weak seed-field is at least one order of magnitude lower than the energy of the strong-field branch.

The magnetospheric feedback is such that magnetopause currents induce an axisymmetric dipole at the CMB that is always anti-parallel to the internal dipole providing a negative feedback on the dynamo, which we use for our implementation. We found that external fields can significantly modify the evolution of the weak seed-field dynamos. In contrast, solutions with strong seed-fields are not significantly affected by the external sources.

We have found that a magnetospheric feedback may be important for dynamos with weak dipole fields and weak convection strength. Such weakened dipole fields may be found for example at dipole polarity reversals. Alternatively, the weak dipole field may have occurred at the initiation of the dynamo (e.g. a non-dipolar seed field). Dynamos of planets with short orbital distances are subject to intense solar winds, and thus they are susceptible to a control of the dynamo field by the magnetospheric feedback.

## **The influence of fixed flux conditions on convections and dynamos**

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The influence of thermal conditions at the core-mantle boundary on the planetary magnetic field has been addressed by several authors. Most focus on the effect of heterogeneous patterns. However, even an uniform fixed temperature or flux condition can affect the structures of flow and magnetic field.

A roll of the flux condition is to enlarge convective patterns in a rapidly rotating system with presence of a magnetic field. This effect may differ between bottom-heated and volumetrically-heated cases. Here we examine the influence of flux conditions on the convective structure in dynamos driven by the different heating modes.

We found that the fixed flux condition at the outer boundary enlarges the convective structures, for dynamos driven by volumetric heating. For bottom-heated dynamos, however, the flux condition at the inner rather than at the outer boundary promotes larger convective cell. Since volumetric heating would be the driving source in cores without an inner core, our results suggest that the early dynamos of Earth and Mars are more sensitive to the thermal boundary condition imposed by the mantle, than the present geodynamo.

## Another 2D code for modeling of two-phase flow: A benchmark and an application in geodynamics

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It has long been recognized that the arc volcanism is directly related to the slab dehydration and water is necessary to explain the low temperature melting of the mantle wedge and possibly of the subducting oceanic crust and the composition of arc magma. To better understand the dynamics of geophysical fluid viz. water, melts in the mantle wedge related to subduction zone, we have developed a numerical model for two-phase flow which takes into account the effects of compaction of the matrix.

In this numerical model, we solve mass and momentum conservation equations of matrix and of fluid following the formulation of Bercovici et al. (2001) using potential formulation of velocities of matrix and fluid. We use several numerical schemes like Finite Difference (FD) method, Successive Over Relaxation (SOR) method to solve potential equations, Multidimensional Positive Definite Advection Transport Algorithm (MPDATA) scheme [Smolarkiewicz et al. (1998)] to solve advection equation i.e. mass conservation equation.

To benchmark our code, we have looked for a solitary wave solution and derived analytical solutions assuming a porosity dependent effective viscosity for the one-dimensional problem along with a method to compute their shapes [Richard et al. (2010)]. Implementing this solution as an initial condition allows us to test our numerical code.

Here we present a formulation of our code and also the results obtained from it.

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## **Numerical modeling of two-phase flow: Interaction of partial melting with active tectonics**

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We investigate the behaviour of a two-phase system that involves partial melt production and percolation through a viscoelastoplastic continental lithosphere and crust under ongoing tectonic deformation. Using two-dimensional numerical simulations we examine the coupled magmatic and tectonic processes leading to intrusive rock formation in a continental setting. We do this by tracking melt from its formation in the upper mantle, during its ascent through the lithosphere, until it is emplaced and crystallized as an intrusive body in the crust.

The numerical modeling approach is based on the assumption that the melt fraction is equal to the porosity of the rock and that porosity change reflects, apart from melting and crystallization, the compaction or dilation of the matrix framework due to both viscous and elastic processes. Both modes of compaction are connected to the local effective pressure, which is obtained as the difference between the bulk pressure over both phases and the local fluid pressure. The magmatic model is chosen to represent a typical melt evolution starting with an arc-type basaltic melt that will fractionate into mafic cumulates and more highly evolved melt, which will again crystallize as a felsic plutonite rock. Compositional contamination by melting of crustal rocks during the magma's ascent is taken into account.

The model setup involves a continental crust of 50 km thickness and 100 km of the underlying mantle. At the lithosphere-asthenosphere transition, we introduce a source region for partial melt by applying an initial temperature slightly above a wet mantle solidus. The melt production and propagation depends on the evolution of temperature and dynamic pressure in the lithosphere and crust as the region is being deformed tectonically. Here, we focus on extensional tectonics as they provide the best conditions for the extraction of mantle melt. Compressional and transpressional tectonics will be the subject of further investigations.

First results indicate that melt propagation is strongly related to the regional stress field, and that brittle fault zones form important conduits for the propagation of partial melt, especially through the more competent parts of lithosphere and lower crust. Where the partial melt reaches either mechanical barriers or neutral buoyancy with respect to the host rock, regions of magma accumulation may quickly evolve into magma chambers with melt content exceeding 80%. There, the melt may either reside until it crystallizes or fractionate until the more evolved rest of the melt has obtained new buoyancy to force its way further through the crust.

A possible application of such models is to deepen the understanding of the processes involved in, and the geometry and field relations expected from, the emplacement of hydrated slab melts into the overriding continental plate in an ocean-continent subduction setting.

# Stokes Solvers for Variable Viscosity Mantle Convection

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Viscosity in the Earth's mantle varies by several orders of magnitude, dependent on temperature, pressure, grain size and phase transformations. Modeling these variations realistically in three-dimensional mantle convection simulations has been a long-standing problem. We present various approaches to increase the robustness and efficiency of the solution process in a 2-D rectangular domain as well as in a spherical shell.

A stabilization of the finite-element discretization with piecewise linear resp. bilinear trial and test functions for both, velocity and pressure, is presented and evaluated in terms of matrix properties of the Schur complement. The stabilization method uses polynomial pressure projections and is described and analyzed in [1]. The resulting discrete Stokes system fulfills a generalized inf-sup condition with a grid-independent inf-sup constant.

Viscosity dependence of the matrix entries in the momentum operator and in the Schur complement is removed by diagonal scaling as is done in [2]. Here, a viscosity scaled pressure mass matrix as in [3] is used. A preconditioner is then necessary only for the momentum operator  $\mathbf{A}$  having a condition number proportional to the number of grid points. This indicates the use of a multigrid method which utilizes the low condition numbers on coarse grids efficiently. Such a method is used in the 3-D spherical code TERRA. For the scaled Schur complement no further preconditioning is necessary so that a CG method can be used to solve for pressure. With applying velocity corrections from the  $\mathbf{A}^{-1}$ -evaluation of the application of the Schur complement, this CG method is named after Uzawa, who gave the idea in [4]. Another idea, taken from [5] is to restart the whole Uzawa algorithm when the  $\mathbf{A}^{-1}$ -evaluation cannot be done with sufficient accuracy. Therewith, and with suitable stopping criteria, which are derived from eigenvalue estimates, the Uzawa method shows impressive robustness w.r.t. viscosity variations and in most cases outperforms a preconditioned MINRES method which has been implemented for comparison. However, as the pressure error is now reduced in a viscosity-dependent norm, induced by the above-mentioned mass matrix, the residual has to be reduced below a lower threshold than in the case of using a standard mass matrix. This drawback is far outweighed by the much better convergence when the viscosity-dependent mass matrix is used.

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# **3D-Temperature-Visualization-Method for Convection-Experiments in the Laboratory**

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During the last years, the possibilities to image the Temperaturefield in Laboratory experiments have greatly improved by using Thermochromic Liquid-Crystals (TLC). Those TLCs have the property to reflect the light of a certain wavelength at a certain temperature via Bragg diffraction. Using different kind of TLCs and illuminating them with a constant wavelength (using a lasersheet), it is now possible to observe different isotherms on a 2D-image within an accuracy of 0.1%. As the obtained images depend on the chosen position for the lasersheet, this method is sufficient for axissymmetrical problems (single-plume experiments) but becomes more difficult to handle for a Rayleigh-Bénard-setup. To get a better understanding of how the spatial pattern evoloves through time, we therefore developped a 3D-Visualization-Method: by scanning rapidly the tank, we can reconstruct a 3D-image of the brightest isotherm.

# Influence of rotation in iron and silicat particles in an early magma ocean

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During its evolution, the Earth most likely experienced a 'Giant Impact' in which a Mars size body hit the early planet. Today it seems widely accepted that the origin of the Moon is a result of this Giant Impact. Another consequence of such an impact would be the formation of a 'Deep Magma Ocean', i.e. a layer of molten material, extending to a depth of about 1000km. Transport of heat and matter in a vigorously convecting Magma Ocean plays a key-role for the further evolution and differentiation of the Earth. The sinking of iron droplets in the convecting Magma Ocean probably provides an effective mechanism leading to the separation of metallic and silicate material. Dense material would finally pond at the bottom of the magma ocean. An instability of this dense material (Rayleigh-Taylor Instability) could lead to a rapid formation of the the Earth's Core. Further, the dynamics of a Magma Ocean under the influence of dense silicate crystals is interesting to study, since it leads to a better understanding of layer formation in the later Earth's mantle. We employed a 3D cartesian numerical model with finite Prandtl number, in order to study the sinking of heavy particles in a vigorously convecting environment. Differently from most approaches we have included the effect of rotation on the flow dynamics. While a significant role of rotation can be ruled out for the today's Earth's mantle, due to the high viscosity of the mantle material, this is not the case for a magma ocean. Our numerical fluid model is based on a Finite Volume discretization, while the numerical model for the iron droplets based on an discret element model for the simulation of granular Material. The particles influence the fluid flow throught the chemical component of the fluid model, wich is the volumetric ratio of the particle in each fluid cell. The particles themselves experience the force of the fluid throught the fluids drag. Also gravitational and coriolis forces act on the particles. In our simulations unlike to other approaches the particles are much smaller than the numerical fluid cells, thus saving computational effort. The parallel model in 3D is capable of modeling fluid-particle systems with over  $10^6$  particles. With the help of computer experiments, using this method, we want to archieve a better understanding of the settling processes in the early magma ocean.

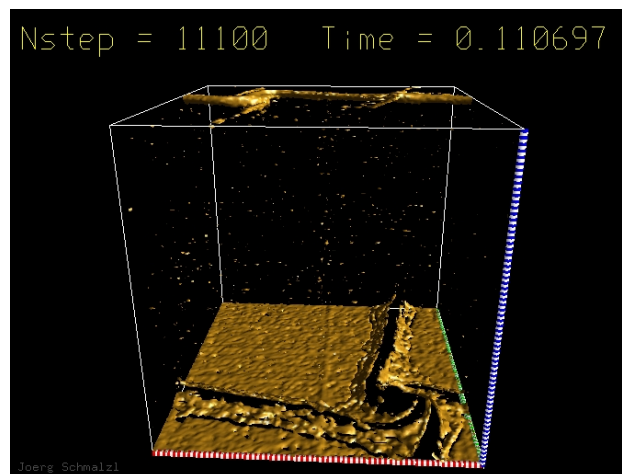


Abbildung 1: First impressions of the behavior of heavy tracers in a convecting and rotating fluid. This picture shows the chemical component, wich is volumetric ratio of the particles in the fluid. The Parameters for this case are  $Ra = 3 \cdot 10^4$ ,  $Ta = 10^4$ ,  $Pr = 1.0$  with over  $1.5 \cdot 10^6$  particles.

# Modelling of the Influence of a Giant Impact and the Resulting Antipodal Anomaly on the Martian Mantle Convection

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## Abstract

We have studied the influence of a large impact on the mantle dynamics of Mars. In contrast to earlier studies, we have also considered the antipodal temperature anomaly that is caused due to the superposition of the shock waves.

## 1. Introduction

Impact processes play an important role in the planetary evolution of terrestrial planets. For instance, it has been suggested that the origin of the Martian hemisphere dichotomy, which is expressed by the change in elevation between the cratered southern highlands and the smooth northern lowlands, is caused by one or several large impacts [1,2,3]. The Hellas basin, which is the largest impact basin on Mars, was formed by an impact during the late heavy bombardment period of the solar system [4]. The massive Hellas strike on one side of Mars is suggested to have triggered the antipodal volcanic region, i.e., Tharsis [5].

The cause for the strong influence of a large impact on the thermal and chemical evolution of the planet is the impact-induced temperature

increase. In a very short time, the mantle material is heated above the solidus and possibly even the liquidus.

Here, we investigate the effects of giant impacts on mantle convection and thermal evolution of Mars in a 2D spherical shell. As a starting temperature condition, we use the results of a 2D impact model. We will show the change of temperature distribution and convection pattern with simulation time.

## 2. Computational Model

For the simulation of the mantle convection, we use the 2D and 3D spherical simulation code *GAIA* [6]. We consider a one-plate planet without a crust. Partial melt is calculated assuming a dry mantle model. The additional temperature in the mantle induced by the giant impact is calculated with the 2D code *iSALE* [7]. We consider an impactor that is perpendicular to the surface.

## 3. Results

The 2D impact model shows that directly after the impact, a shock wave leads to a strong



heating from the impact basin to the interior. The mantle temperature increases due to the high induced pressure. In a very short time, the mantle material is heated above the solidus (and liquidus) curve, such that a high amount of melt is produced and reaches the surface, until the mantle temperature sinks again below the solidus. Still, the mantle temperature is much higher than the temperature of the surrounding material.

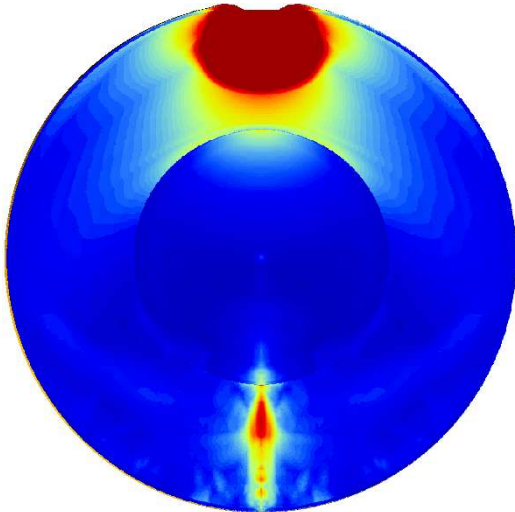


Figure 1: Temperature profile after giant impact.

Depending on the velocity and mass of the impactor, not only the material next to the impact basin heats up. Due to superposition of the shock waves, which reach the opposite site of the mantle and additional waves, which are

reflected from the surface, a pipe-like positive temperature anomaly can be seen at the opposite side of the impact, see Figure 1.

The influence of this particular temperature distribution on the mantle dynamics has been compared with a model assuming just a one-sided temperature anomaly.

Also the influence on the convecting mantle has been investigated in two different scenarios where the impact region was located above an upwelling and downwelling plume respectively.

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# Damped Frank-Kamenetskii Approximation for Temperature- and Pressure-Dependent Rheologies and Consequences for the Simulation of Super-Earths

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The viscosity of a silicate mantle is strongly dependent on temperature and viscosity and can be described by the Arrhenius law [Karato and Wu 1993]. Assuming realistic values of the activation energy and volume, viscosity variations in a terrestrial mantle amount to values of about  $10^{35}$  Pas or larger. Most numerical codes, however, can only deal with much smaller viscosity contrasts. A common method to reduce the viscosity contrast is to use the Frank-Kamenetskii approximation [Frank-Kamenetskii 1969]. Applying this approximation for purely temperature-dependent viscosity, the results are similar to those using the Arrhenius law when the convection is in the steady-state stagnant lid regime [Solomatov and Moresi 1996; Plesa et al. 2010].

Here, we compare the Frank-Kamenetskii approximation to the Arrhenius law for both temperature- and pressure-dependent viscosity. For this, we have derived new Frank-Kamenetskii parameters that also include the pressure-dependence of the viscosity and differ from previous approximations [Christensen 1984, Hansen and Yuen 2000]. We show that using these parameters, the depth-dependence of the approximated viscosity is comparable to the more realistic Arrhenius viscosity. It is even possible to model a stagnant lower mantle that may form above the core-mantle boundary in case of high activation volume or high pressure like for massive Super-Earths [Noack et al. 2010].

Nevertheless, for high surface temperatures like for Venus this approximation does not represent the mantle flow as it is derived by the Arrhenius law. In these cases no stagnant lid forms with the classical Frank-Kamenetskii approximation because the linearized viscosity results in a viscosity contrast of less than  $\sim 10^5$ . For a viscosity contrast lower than  $\sim 10^5$  the convection regime changes to the transitional or mobile regime [Solomatov and Moresi 1996]. To overcome this problem, we have further derived a new approximation, which we call the Damped Frank-Kamenetskii approximation. This is a mixture between the classical law and a second-order approximation controlled by a damping parameter. The second-order approximation is not a linearization of the exponential viscosity term like for the classical Frank-Kamenetskii approximation, but a quadratic approximation with a higher accuracy. The new method leads to a stagnant lid in all cases treated in our investigations. Note that for planets with low surface temperatures, this new approximation can be used as well with a damping parameter of zero that yields the standard Frank-Kamenetskii approximation.

Our studies suggest that the classical Frank-Kamenetskii approximation with the here derived parameters can be used to simulate the mantle dynamics in Super-Earths even with a high pressure-dependence of the viscosity if the viscosity contrast is above  $\sim 10^5$  and a stagnant lid forms on top the convecting mantle. In case the constraint of a stagnant lid is not satisfied, the new Damped Frank-Kamenetskii approximation should be used instead.

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# Numerical Modelling of the magmatic crust production in Iceland

## Abstract

By Robert Orendt

In my Bachelor-Thesis I intended to create 2D- models of the development history of the icelandic crust using the Fortran – based programming code FDCON and compare the results with the empirical data of the crustal thickness obtained by seismic measurements. FDCON solves finite differential equations in a 2-dimensional grid with variable time step width and number of grid points to calculate new values for the density, temperature, viscosity fields using the previous. Equations like mass-, energy and momentum conservation are leading to the biharmonic equation, which is solved in dependence of the stream function. The Bousinesq – approximation neglecting density differences by compaction is in use. All layers are represented by incompressible fluids. The numerical stability is guaranteed by the Courant-criterion. First of all, basing on the paper *Crustal accretion and dynamic feedback on mantle melting of a ridge centred plume: The Iceland case* by Schmeling/Marquardt 2008, it was assumed that all magma which is produced beneath the crust is extracted directly to the surface by volcaneous activity. This lead to the case of *mantle wedge surfacing*, meaning that there was no crust production directly above the mantle plumes, but strong crust thickening to a maximum of 30km with larger distance from the plumes, which is believed to be situated in the mid- atlantic rift directly under the Vatnajökull volcano. Basing on this model, various parameters like the intrusional mass percentage and depth, the plumes temperature and velocity was varied leading to a more realistic case of a *weakly thickening crust*. The crustal thickness amounted to 25km in the plume area and 30 km in the distance, effective for a plumes temperature of 1484°C. For a plumes – temperature of 1634°C both the crustal thickness and expansion velocity of the crust was significantly higher, with a maximum of 40km thickness and 2 cm half – spreading velocity of the plate. In contrast, the seismic measurements indicate a maximum crustal thickness of 40 km directly over the plumes. One of the biggest challenges of numerical modelling will be to bring the results of the computational models in accordance with the data obtained by geophysical measurements.

# Investigation of Flow Reversals in Vigorous Rayleigh-Bénard Convection

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## Abstract

Experimental and numerical studies of thermal convection have shown that sufficiently vigorous convective flows exhibit a large-scale thermal wind component sweeping along small-scale thermal boundary layer instabilities. A characteristic feature of these flows is an intermittent behavior in form of irregular reversals in the orientation of the large-scale circulation. There have been several attempts towards a better understanding and description of the phenomenon of flow reversals, but so far most of these models are based on statistical analysis of few point measurements or on theoretical first order assumptions.

The analysis of long term data sets ( $> 10^7$  time steps) gained by numerical simulations of turbulent 2D Rayleigh-Bénard convection enables us to get a more comprehensive view on the spatiotemporal flow behavior.

By means of a global statistical analysis of the characteristic spatial modes of the flow we are able to extract information about the stability of dominant large-scale modes as well as the reversal path in phase space. We examine PDF's and drift functions of two dimensional phase spaces spanned by different large-scale spartial modes. This also provides information about the coexistence of dominant modes.

# The Influence of Partial Melt Generation on Mantle Density and Viscosity: Consequence for the Mantle Dynamics

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Melt generation in a planetary mantle is a complex process that has a strong influence on the thermochemical evolution of terrestrial planets, being present during some period of planetary evolution [Solomatov and Moresi, 2000]. In most earlier thermal evolution and convection models, melt generation has been considered by the consumption and release of latent heat, the associated formation of crust and the redistribution of radioactive heat sources [e.g. Hauck and Phillips, 2002; Breuer and Spohn, 2003; Schumacher and Breuer, 2006].

However, when modeling partial melt it is important to consider also the effects on mantle (1) density and (2) viscosity. Here we discuss the influence of partial melt on mantle density and viscosity assuming fractional melting, i.e., melt leaves the system as soon as it is formed, and a wet planetary interior as it has been suggested for the Martian mantle [e.g., Hauck and Phillips, 2002; Grott and Breuer, 2008].

First, the density of the mantle material decreases with increasing degree of depletion due to compositional changes. The extraction of partial melt leaves behind a residuum depleted in incompatible elements and modified in modal mineralogy, which is expected to be more buoyant than its fertile parent material [e.g., Morgan, 1968; Schutt and Leshner, 2006]. This process can lead to the formation of a buoyant upper mantle, having a stabilizing effect on the mantle dynamics and preventing efficiently the planet from cooling.

Second, melt can also indirectly impact the viscosity of partially molten rocks through its influence on the water content [Hirth and Kohlstedt, 1996]. Mantle material will be dried out due to partitioning of water from the minerals into the melt during the melting process. As a result, the viscosity of water depleted regions increases more than two order of magnitude compared to the water-saturated rocks [Korenaga, 2009].

Using a 2D spherical convection model that can handle radial and lateral variations in the viscosity [Hüttig and Stemmer 2008a; Hüttig and Stemmer, 2008b], we investigate systematically the effects of partial melt on mantle dynamics and thermal evolution of a one-plate planet. We assume Mars-like parameters, a cooling core boundary condition and decaying heat sources.

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## How to form a Basal Magma Ocean? Insights from two-phase flow numerical modeling

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In a recent paper Labrosse et al. (2007) have proposed that the sparse ultra low velocity zone observed at the base of the Earth's mantle, and generally interpreted as patches of dense partial melt (Williams & Garnero, 1996), could be the vestiges of a basal magma ocean once overlying the core mantle boundary.

To investigate the physical mechanisms involved in the formation of such a basal magma ocean, we have designed a two-phase flow model describing the early mantle of the Earth as a mixture of melt and viscously deforming solid matrix. More specifically our model takes into account the compressibility of melt with depth and the melting of the matrix via a coupling source term.

Because of its compressibility the melt eventually becomes denser than the surrounding matrix. Consequently, above this critical density cross-over depth, the melt is percolating upwards to form a magma ocean at the surface while symmetrically below this depth it is migrating downward to form a basal reservoir. Meanwhile the rocky matrix deforms as well inducing compaction and thermal adjustment.

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## **Influence of continents in mantle convection models with self-consistent plate tectonics**

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It is now well accepted that mantle convection and plate tectonics form an integrated system and cannot be treated independently. Although this is a promising improvement in understanding Earth, there is still a striking feature, which is nowadays not yet included in this integrated system, namely the existence of a lithospheric heterogeneity - in other words - the difference between oceans and continents.

The present study focusses on the effect of continents in a model of self-consistent plate tectonics in spherical geometry. As a simplification these continents are realized as strong cratons with homogeneous composition and they differ from the rest of the mantle in buoyancy and rheology. In contrast to many former studies where continents are idealized as rigid and/or immovable units, we treat continents in the same manner as normal mantle, but with different physical properties. Numerically a tracer approach is used, which allows more consistent movement and deformation of the continents.

It has been shown before that continents might have a first-order effect on the dynamics of the Earth as they might modulate convective wavelength, surface heat loss and - due to thermal insulation - the internal temperature. Increasing the latter causes a decrease in convective stresses and we studied how this effect strengthens the lithospheric lid, what can finally lead to a transition from mobile lid to stagnant lid convection. Existence and timescale of this transition depend on the initial strength of the lithosphere and are most sensitive to internal temperature variations, but less sensitive to relative continental buoyancy.

The mentioned transition will be studied into more detail. A question of particular interest is, if the system behaviour changes, if the continents are no longer embedded in the thermal boundary layer. For answering this question it is necessary to modify the rheology of the continental material, namely to consider a viscosity, that depends on composition, and to make sure that continents are initially cold.



# The Numerical Advection of Discontinuous Quantities in Geophysical Flows Using Particle Level Sets

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Advection a major process that commonly acts on various scales in nature (core formation, mantle convective stirring, multi-phase flows in magma chambers, salt diapirism ...). While this process can be modeled numerically by solving conservation equations, various geodynamic scenarios involve advection of quantities with sharp discontinuities. Unfortunately, in these cases modeling numerically pure advection becomes very challenging, in particular because sharp discontinuities lead to numerical instabilities, which prevent the local use of high order numerical schemes.

Several approaches have been used in computational geodynamics in order to overcome this difficulty, with variable amounts of success. Despite the use of correcting filters or non-oscillatory, shock-preserving schemes, Eulerian (fixed grid) techniques generally suffer from artificial numerical diffusion. Lagrangian approaches (dynamic grids or particles) tend to be more popular in computational geodynamics because they are not prone to excessive numerical diffusion. However, these approaches are generally computationally expensive, especially in 3D, and can suffer from spurious statistical noise.

As an alternative to these aforementioned approaches, we have applied the relatively recent Particle Level set method for modeling advection of quantities with the presence of sharp discontinuities. We have tested this improved method, which combines the best of Eulerian and Lagrangian approaches, against well known benchmarks and classical 2D and 3D Geodynamic flows.

In each case the Particle Level Set method accuracy equals or is better than other Eulerian and Lagrangian methods, and leads to significantly smaller computational cost, in particular in three-dimensional flows, where the reduction of computational time for modeling advection processes is most needed.

## Thermo-mechanical modelling with a free surface: the sticky air approach

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Numerical thermo-mechanical models of the crust or lithosphere often need to take into account the free surface in order to capture the formation of dynamic topography. Dynamic topography may be important as an observable quantity of dynamic processes such as rifting, as a feedback into the dynamics of the model, e.g. for subduction dynamics, as a significant contribution to the geoid; and as an important contribution to the total topography.

The boundary condition of a free surface in a dynamic model is zero traction at the (deformed) surface. In Lagrangian approaches such as Finite Elements this condition is easily fulfilled, however care has to be taken, to ensure that the mesh is advected with the displacement or flow. In Eulerian approaches usually regular grids are used. Thus precise tracking of the free surface is necessary. Two alternative approaches are usually used:

a) Free slip, no vertical movement at top of the model (*McKenzie, 1974*): The normal stress  $\sigma_{zz}$  at the surface due to the flow is taken to obtain the topography  $h$  using the first term of a Taylor series expansion, i.e.  $\sigma_{zz} = -\rho g h$  ( $\rho$  - density,  $g$  - gravitational acceleration). This works well if  $h \ll$  wavelength and if the slopes remain small. As a disadvantage also the pressure is needed, which may cause numerical resolution problems for large viscosity contrasts. Another drawback is the assumption of instantaneous isostatic adjustment, which may cause spurious effects for time-dependent processes.

b) Sticky air layer: In this approach the model box is vertically enlarged by a thin layer of low viscous material of density zero (*Schmeling et al., 2008*). To properly resolve topography variations on an isostatic relaxation time scale, a condition for the thickness  $h_{st}$  and the viscosity  $\eta_{st}$  of the sticky air layer has to be fulfilled, namely  $C \ll 1$  with

$$C = \frac{3}{8\pi} \left( \frac{L}{h_{st}} \right)^3 \frac{\eta}{\eta_{st}} \quad (1)$$

where  $L$  = width of the model,  $\eta$  = characteristic model viscosity. In the long term isostatic limit  $C$  may be replaced by

$$C = \frac{1}{16} \frac{\Delta\rho}{\rho} \left( \frac{h_{model}}{h_{st}} \right)^3 \frac{\eta}{\eta_{st}} \quad (2)$$

with  $h_{model}$  = model thickness. In both cases, usually the time step has to be below the isostatic relaxation time. Problems encountered by this approach include: Entrainment of sticky air in subduction modelling, thermal convection within the sticky air layer, numerical erosion at the model – sticky air interface.

Comparison of the different approaches show: In rift models viscous bending of the lithosphere with a few 100 m flank uplift is absent (approach a) or present (b). Sticky air captures isostatic uplift on a kyr scale, which is absent for the stress derived topography. In conclusion, fully free surface cases and sticky air cases agree well if the viscosity and thickness of the sticky air layer fulfill the above mentioned condition with  $C$  given by (2).

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## **Numerical modeling of granitic intrusion mechanisms**

(Maike Schubert, Thomas Driesner, Taras Gerya, ETH Zürich, Switzerland)

The idea of generation of granitic magma by partial melting of lower crust due to temperature anomalies is well accepted. These anomalies can be caused either by intrusions of mantle-derived magmas or by underplating of the lower crust by hot asthenospheric material. However, it is still not clear which physical mechanisms are responsible for this magma to rise and form intrusions in the middle and upper crust.

In order to understand magma ascent and emplacement processes we used a 2D Cartesian, visco-elasto-plastic, finite difference numerical model (code I2ELVIS) with a length of 1100km and a depth of 200km. The model is self-consistent and includes source regions for both crustal and mantle magmas.

We assume a magma reservoir of mafic magma in the sub-lithospheric mantle which is connected with the bottom of the lower crust by a pre-defined vertical magmatic channel. Starting with a positive temperature-anomaly in the lower crust we study the ascent and emplacement of granitic magma in the upper crust. Results of numerical experiments suggest that crustal magma rising is triggered by a spontaneous increase of overpressurized mafic magma from the sublithospheric source into the region of the crustal temperature anomaly. Depending on the rheological properties of the crust, the amount and temperature of the lower crustal magma it either erupts to the surface or intrudes into the upper crust. Magma chambers of different size and shape develop in the upper and lower crust.

In a next step we plan to implement more realistic properties of the magmas, in particular a temperature-pressure-composition-dependent rheology and include processes like degassing of the magma due to lower pressure at shallower depth. Degassing and cooling of the magma chamber will be studied at high resolution on a magma chamber scale.

# Initial Convection Experiments in Spherical Shells for Modelling Mantle Dynamics within the GeoFlow project

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Thermal convection is a central objective in geo- and astrophysical research. To model convection by an experiment we consider the fluid motion in a gap between two concentric spheres, with inner spherical shell heated and outer spherical shell cooled. Central symmetry buoyancy field is set-up by means of a high voltage potential and use of a dielectric insulating liquid as working fluid in the spherical cavity. This technique, i.e. realization of a selfgravitating force field experimentally, requires microgravity conditions in order to reduce unidirectional influence of gravity, that would dominate fluid flow in the Earth laboratory.

For GeoFlow these specific conditions are available in the European COLUMBUS module part of the International Space Station ISS [1]. During GeoFlow I mission, which was running on orbit from July 2008 until January 2009, the shells were filled with a silicone oil with approximately constant viscosity. Motivated by convective motion of the Earth's outer core, patterns of convection and their spatial-temporal behaviour have been prospected. For the planned second GeoFlow II mission (on orbit 2010) we propose to use 1-Nonanol as the working fluid, having a temperature dependent viscosity. Therewith experimental modelling of mantle convection is the central goal [2].

Governing equations in Boussinesq form for the incompressible Newtonian fluid of 1-Nonanol are dominated by inertia. In contrast to traditional computer simulation work for Earth mantle dynamics the Prandtl number for our planned experiment is reasonable high ( $Pr \leq 200$ ), but not infinite. Therefore in a first step this Prandtl number influence has been benchmarked with the spherical code GAIA assuming an isoviscous fluid and set-up with an infinite Prandtl number. As a conclusion from these numerical tests, the Prandtl number can be dropped. There steps are to simulate variations of thermal forcing (variation of Rayleigh number) with the specific viscosity contrast of 1-Nonanol.

With further tests for the overall behaviour of GeoFlow II's experimental fluid 1-Nonanol, it shall be possible to evaluate the theoretical predictions on thermal, dielectric and optical performance of the fluid. Two aspects in realization the experimental runs have to be considered: Increasing the viscosity contrast accompanied by decreasing  $Ra$  and vice versa. For this the working environment of the experiment has to be varied. Images of the tested runs as well as reached viscosity contrast demonstrate very clearly that the experimental fluid acts in combination of variation of working environment and variation of temperature difference.

## Planetary Evolution and Habitability

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Planetary habitability is usually thought to require water on (or near) the surface, a magnetic field to protect life against cosmic radiation, and transport mechanisms for nutrients. A magnetic field also serves to protect an existing atmosphere against erosion by the solar wind and thus helps to stabilize the presence of water and habitability. Magnetic fields are generated in the cores of the terrestrial planets and thus habitability is linked to the evolution of the interior. Moreover, the interior is a potential source and sink for water and may interact with the surface and atmosphere reservoirs through volcanic activity and recycling. On the Earth, water is stabilized by complex interactions between the atmosphere, the biosphere, the oceans, the crust, and the deep interior. The most efficient known mechanism for recycling is plate tectonics. Plate tectonics is known to operate, at present, only on the Earth, although Mars may have had a phase of plate tectonics as may have Venus. Single-plate tectonics associated with stagnant lid convection can also transfer water from the interior but a simple recycling mechanism is lacking for this tectonic style. Stagnant lid convection will evolve to thicken the lid and increasingly frustrate volcanic activity and degassing. (This can keep the interior from running completely dry.) Plate tectonics also supports the generation of magnetic fields by effectively cooling the deep interior. (In addition, plate tectonics rejuvenates nutrients on the surface and generates granitic cratons.) For Mars and Venus it is likely that a present-day magnetic field would require plate tectonics to operate. An early field is possible even with stagnant lid convection but the dynamo will only operate less than about a billion years. This dynamo would have been driven by thermal buoyancy and require that the core was sufficiently superheated with respect to the mantle after core formation. The dynamo would have ceased to operate as the core cooled depending on the vigor of mantle convection. A question is then whether or not plate tectonics existed on Mars and Venus and if yes why plate tectonics ceased to operate. Or, more generally, why do planets have plate tectonics and others do not? Convection model calculations suggest relations to the yield strength of the mantle and the effect of water on the latter. Other models suggest that the existence of an asthenosphere (a low viscosity zone underneath the lithosphere) may be decisive. The presence of water will lower the solidus of mantle rock and help to form an asthenosphere. Thus, there appear to be links between plate tectonics and (near) surface water, plate tectonics and magnetic fields, magnetic fields and habitability, and habitability and water. Is plate tectonics even a potential biosignature?

# On the Problem of the Propensity of Plate Tectonics on Super-Earths

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The last decades of astronomical observation have opened the new interdisciplinary field of extra-solar planetary research. Up to this date around 500 exoplanets were detected, most of them with masses in the range of Jupiter. With improved technology the mass detection limit has been reduced to planets consisting of a few Earth masses. Planets with a structure similar to Earth but being more massive are called Super-Earths, such as Corot-7b (approx  $5M_E$  and  $1.6R_E$ , [Queloz et al. 2009](#)). Recently, questions have been raised about the ability of Super-Earths to sustain active plate tectonics, similar to Earth. [Valencia et al 2007](#) and [O'Neill and Lenardic 2007](#) have investigated the ability of Super-Earths to overcome lithospheric stresses and to deform plates, but both derived opposite conclusions.

Both models were using a pressure-independent rheology, which is not suitable to describe the mantle convection of Earth and Super-Earths as shown in [Stamenkovic et al 2010 a\)](#) and [b\)](#).

Nonetheless to test the robustness of their findings, we at first investigate a pressure independent rheology (with activation volume  $V^*=0$ ), using a parameterized 1D thermal boundary layer model similar to [Valencia et al 2007](#). We define a driving propensity of plate tectonics for every planet in relation to a test planet with one Earth mass for every time step. The driving propensity describes for every planet at a given time the ratio of convective stresses versus the yielding stresses below the planetary lid in relation to a one Earth mass planet. We test the sensitivity of the results in dependence to the choice of the two main scaling parameters of the model, namely the Nusselt-Rayleigh scaling exponent  $\beta$  and the scaling Rayleigh number  $Ra$  ( $Nu \propto Ra^\beta$ ).

For isoviscous convection boundary layer theory predicts that  $\beta$  depends on the type of heating (internal or bottom) and on the boundary conditions (free-slip or no-slip). Ideally, in the case of free-slip conditions,  $\beta=1/4$  for internal, and  $\beta=1/3$  for bottom heating respectively. 3D and 2D calculations have shown that for isoviscous and non-isoviscous convection with varying boundary conditions and mixed heating  $\beta < 1/3$  and is actually closer to  $\beta \approx 1/4$  for any kind of heating mode ([Choblet and Parmentier, PEPI 173, 2009](#); [Iwase and Honda, GJI 180, 1997](#); [Sotin and Labrosse, PEPI 173, 1998](#)). We observe that for  $V^*=0$  we only obtain increasing driving propensities with increasing planetary mass for Super-Earths, as observed by [Valencia et al 2007](#), if  $\beta > 0.32$ , which is not realistic for mixed heating.

Parameterized Boundary Layer Models use the Rayleigh number as their scaling parameter. The Rayleigh number  $Ra$  depends on values that change with depth in planetary mantles, which we derive in [Stamenkovic et al 2010 a\)](#) for phononic contributions:

- Density **increases** with depth
- Thermal expansivity **decreases** with depth

- Thermal diffusivity **increases** with depth

The Rayleigh number is used to scale the whole depth-dependent, compressible system. Note that *Valencia et al 2007* has inconsistently used an upper mantle thermal expansivity and an upper mantle thermal diffusivity but kept an average mantle density. For a depth-dependent system with “local” Rayleigh numbers strongly varying it comes to hand to describe the convective system with a Rayleigh number defined with all average mantle values. In this case, the driving propensity decreases for more massive Super-Earths. We further find that in an as well consistent model with only upper mantle values, the driving propensity would remain almost unaffected by the planet’s mass.

In both cases we can show that the results obtained by *Valencia et al 2007* are crucially depending on the unrealistic model assumptions and by far not robust. For Super-Earths our results tend to agree well with the findings of *O’Neill and Lenardic 2007*, where the driving propensity of plate tectonics is decreasing with planetary mass, but our results disagree for planets smaller than Earth, where we observe a lower driving propensity at all times and for the whole parameter range in comparison to a one Earth mass reference planet.

The results so far were for  $V^*=0$ , which is not realistic for planets as large as or larger than Earth. In *Stamenkovic et al 2010 a)* we derived a more realistic viscosity law, suitable for Earth and Super-Earths with  $V^*\neq 0$ . We discuss the implications of this pressure-dependent viscosity for our results and show that it most likely leads to a further decrease of driving plate tectonics propensity on Super-Earths, as already observed for  $V^*=0$ .

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## A geodynamic model of plumes from the margins of Large Low Shear Velocity Provinces

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Using plate tectonic reconstructions in an absolute mantle reference frame, it has been shown that the eruption sites of most Large Igneous Provinces as well as kimberlites during the last few hundred Myr lie - like many present-day hotspots - above the margins of the two Large Low Shear Velocity Provinces (LLSVPs) of the lowermost mantle. This indicates that plumes preferably get created at these margins, and that LLSVPs locations are rather stable for at least 200 Myr, and further supports the notion that the LLSVPs are chemically distinct from, and heavier than the rest of the mantle.

Here a geodynamic model that can explain this characteristic of the Earth's mantle is presented. Time-dependent density distribution of the Earth's mantle is modelled based on 300 Myrs of subduction history, with a spherical harmonic code and a radial viscosity structure constrained by mineral physics and surface observations. The initial condition also features a heavy chemical layer (3.2 % density anomaly) of 70 km thickness at the base of the mantle. Thermal density anomaly at the CMB is 2 % and thermal diffusivity about  $10^{-6} \text{ m}^2/\text{s}$ . The sinking subducted slabs form the chemical layer to two distinct large piles under Africa and the Pacific whose location approximately coincides with the two LLSVPs, as well as a smaller pile under Siberia, which may correspond to a smaller Low Shear Velocity Province. They also push the thermal boundary layer towards the chemical piles. Once this hot material reaches the steep edges of these piles, it is forced upwards and begins to rise - in the lower part of the mantle as sheets, which then split up into individual plumes in the upper mantle. Each pile is thus crowned by 4-5 plumes, sitting like candles on a birthday cake, while a separate plume rises under Siberia. Plume conduits become tilted with their bases moving towards the centers of the piles, while their tops remain over the margins.

Due to high viscosities in the lower mantle up to about  $10^{23} \text{ Pas}$ , plumes in our model are rather massive (diameters  $> 500 \text{ km}$ ) and entrain a substantial part of the chemical layer over the time of the model run. Future models with lateral viscosity variations will aim at maintaining the creation of plumes at the margin of piles, while additionally reducing entrainment and thus enabling longer-term stability of chemically distinct piles



## **Lower mantle dynamics and the role of pressure-dependent thermodynamic and transport properties**

*Nicola Tosi, David Yuen and Ondrej Cadek*

We have carried out numerical simulations of large aspect-ratio 2-D mantle convection that feature pressure-dependent thermal expansivity and conductivity along with the major mantle phase transitions, including the deep phase change from perovskite (pv) to post-perovskite (ppv). The rheological law is Newtonian and has both temperature- and pressure-dependences, while the extended Boussinesq approximation is assumed for the energetics. We have analyzed the combined effects of a strongly decreasing thermal expansivity, according to the diffraction experiments on pv by Katsura et al. (2009), and steeply increasing lattice thermal conductivity based on different models obtained from experiments (Ohta, 2010) and first principles (de Koker, 2010; Tang and Dong, 2010). Since ppv is expected to have a relatively weak rheology with respect to pv (Hunt et al., 2009; Ammann et al., 2010) and a large thermal conductivity (Ohta, 2010), we have also assumed that the transition from pv to ppv is accompanied by both a reduction in viscosity by 1 order of magnitude and by a 50% increase in conductivity.

As long as the thermal expansivity and conductivity are constant, ppv exerts small but noticeable effects: it destabilizes the D" layer, causes focusing of the heat flux peaks and a slight increase of the average mantle temperature and of the temporal and spatial frequency of upwellings. The destabilizing character of ppv is strong enough to affect the stability of mantle plumes even in the presence of a large decrease of the thermal expansivity which otherwise, without ppv, delivers remarkably stable large upwellings. However we have found that if a sufficiently large thermal conductivity near the core-mantle is also accounted for, lower mantle plumes are stabilized for a geologically long time-span in excess of billion of years, even in the presence of the disturbances induced by the pv-ppv transition. Preliminary results confirm the validity of these findings even for thermo-chemical piles in the framework of thermo-chemical convection with important implications for the understanding of the large low shear velocity provinces (Dziewonski et al., 2010; Torsvik et al., 2010).

The combination of strongly depth-dependent expansivity and conductivity is a viable mechanism for the formation of long-wavelength, long-lived thermo-chemical anomalies in the deep mantle, even if a low-viscosity ppv atop the core-mantle boundary is included.

# Numerical simulations of thermo-chemically driven convection and geodynamos

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Our numerical study focuses on convection and magnetic field generation in a rotating spherical shell with the objective to model combined thermal and compositional convection as proposed for the Earth’s core. Since the core of the Earth is cooling, a thermal gradient is established, which can drive thermal convection. Simultaneously, the advancing solidification of the inner core releases latent heat and increases the concentration of the light constituents of the liquid phase, e.g., sulphur, oxygen, and silicon, at the inner core boundary [Fearn, 1998]. Thus, buoyancy is created by both thermal and chemical heterogeneities. Typically, the molecular diffusivities of both driving components differ by some orders of magnitude [Braginsky and Roberts, 1995] indicating that one has to consider two separate transport equations in the numerical solution. In our double-diffusive convection model, we assume that the thermal diffusivity  $\kappa_T$  exceeds the compositional one  $\kappa_C$  by a factor of ten ( $\kappa_T/\kappa_C=10$ ). The core mantle boundary is supposed to be impermeable for the light component. The freezing inner core, however, provides a certain flux of light material at the inner core boundary. Therefore, appropriate Neumann boundary conditions are implemented in the numerical scheme. The ratio of thermal to chemical forcing in the Earth’s core is still rather uncertain. As a joint action of both buoyancy sources is most likely we investigated core convection in a range of varying thermal to chemical forcing ratios. We find that the patterns of spatial flow structures like differential rotation and helicity depend significantly on the particular driving scenario (see fig. 1(a) and 1(c)). Additionally, we compare our results to equivalent simulations with Dirichlet boundary conditions thus assessing the influence of the different types of boundary conditions on the convective flow. Furthermore, we investigated the effect of different thermo-chemical driving scenarios on the process of magnetic field generation of the MHD geodynamo. It turns out that the distribution of magnetic energy inside and outside the tangent cylinder depends significantly on the thermal to chemical forcing ratio (see fig. 1(b) and 1(d)).

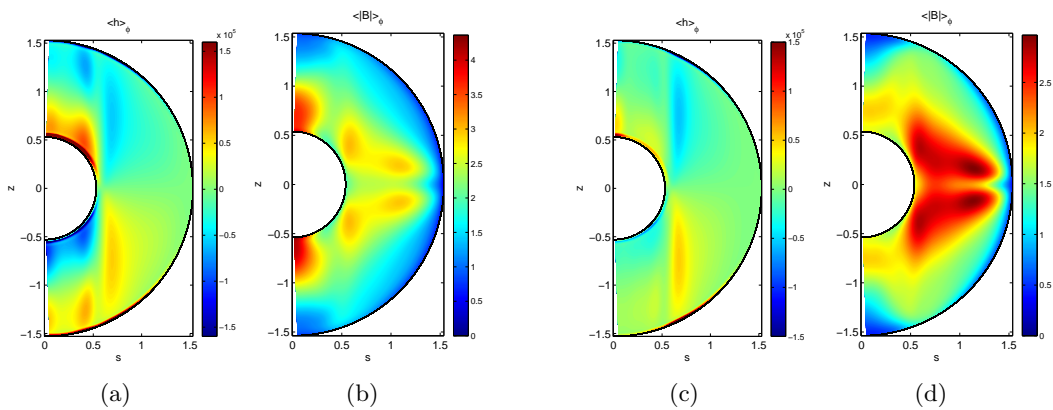


Figure 1: Temporally and azimuthally averaged helicity  $h = \vec{u} \cdot (\nabla \times \vec{u})$  and total magnetic field  $|\vec{B}|$  of two simulations with different driving scenarios. Predominantly chemically driven: (a) and (b). Chemical and thermal driving of equal strength: (c) and (d).

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## Numerical modelling of the plate retreat at the Hellenic Subduction Zone

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### Abstract

Subduction is a tectonic process, where one tectonic plate of oceanic lithosphere moves under another tectonic plate of usually continental lithosphere and sinks into Earth's mantle. A subduction zone is the area, where subduction takes place and is characterised by deformation. As the oceanic plate subducts an arcuate trench is shaped between the two plates. The deformation of the upper plate is either tensional or compressive.

Regarding the Mediterranean region the deformation was mainly controlled by slab retreat for the last 30-35 Ma [1]. Slab retreat means, that the subducting slab migrates away from the upper plate and not towards it. The trench at the Hellenic Subduction Zone is strongly curved and the upper plate is stretched at the Aegean.

With the intention to do a three-dimensional finite element model for reproducing the deformation of this region first a two-dimensional plane strain finite element model is set up as a start for this. Mainly this is done to get a first information about the conditions for slab retreat by a simple model and by avoiding the computational effort related to the three-dimensionality. The two-dimensional model includes a plane strain box with a portion of mantle, the two interacting plates and a top light weak layer representing air or water. An initial dip for the subducting plate is assessed. All boundaries except the top one are free-slip boundaries, the top boundary is free. The applied load is gravity load, no velocities are imposed. A Maxwell-viscoelastic incompressible material is chosen with varying relaxation times for the different layers. For this setup the advance of the plate retreat is monitored. The information of the two-dimensional models is then included in the three-dimensional model.

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# Extension of a Cartesian Geometry MHD Boussinesq Code to the Anelastic Approximation

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## Abstract

Most numerical simulations use the framework of the Boussinesq approximation for modeling the turbulent convective flows in the planetary interiors of our solar system. Typically, the planets' density varies considerably with radius. Such background density variations have a large impact on the interior flow dynamics and are completely neglected in Boussinesq models. However, the effects of density stratification can be studied by using the anelastic approximation which - in contrast to fully compressible models - filters out fast time scales caused by sound waves.

In our anelastic approach we assume a perfect gas with the heat diffusion depending on the entropy instead of the temperature. Due to eliminating the pressure, the entropy is left as the only thermodynamic variable in the set of time-dependent equations. Furthermore we concentrate on the special case of constant dynamic viscosity while the entropy's turbulent diffusion coefficient is assumed inversely proportional to density keeping the Prandtl number constant.

As the governing equations of the anelastic system are very similar to the Boussinesq ones, we extent an existing Cartesian geometry spectral Boussinesq code to our anelastic model. In the horizontal direction the unknowns are expanded into truncated Fourier series. To allow for a vertical density stratification we apply finite difference formulas of arbitrary order of accuracy on grids with arbitrary spacing. The new models' code is kept very general. Consequently it can be used for 2D/3D and Boussinesq/anelastic simulations with different time stepping schemes.

So far we we successfully reproduced the results of a benchmark suggested by Lantz and Fan (The Astrophys. J. Supp., 121: 247-264, 1999) that tests the code's nonlinear dynamics. Further benchmarks are currently in progress.

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# **Geodynamic regimes of crustal growth at active margins: numerical modeling**

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The dynamics of crustal growth under active continental margins were analyzed by using a coupled 2D petrological-thermomechanical numerical model of an oceanic-continental subduction process. This model includes spontaneous slab retreat and bending, dehydration of subducted crust, aqueous fluid transport, partial melting, melt extraction and melt emplacement in form of both extrusive volcanics and intrusive plutons. Depending on variable model parameters such as plate velocities and degree of rheological weakening induced by fluids and melts, the following three geodynamic regimes of crustal growth were identified: (i) stable subduction without plume development (ii) subduction associated with plume emplacement and (iii) subduction accomplished by lithosphere extension and back arc spreading. Crustal growth in a stable subduction setting results in the emplacement of flattened intrusions within the lower crust of mainly basaltic to andesitic composition. At first melts extracted from partially molten rocks located atop the slab (i.e. hydrated mantle, sediments and basalts) intrude into the lower crust followed by mantle-derived (wet peridotite) basaltic melts from the mantle wedge. Thus, extending plutons form associated with low crustal growth rates ( $15 \text{ km}^3/\text{km}/\text{Myrs}$ ) and a successively increasing mantle component. In a plume-present regime, crustal growth is accomplished by the formation and emplacement of silicic plumes. In the course of subduction localization and partial melting of basalts and sediments along the slab induces Rayleigh Taylor instabilities. Hence, buoyant silicic plumes are formed, composed of partially molten sediments, basalts (oceanic crust) and serpentinite. Subsequently, these plumes ascend, crosscutting the lithosphere before they finally crystallize within the upper crust in form of silicic batholiths. Additionally, basaltic intrusions within the lower crust are formed derived by partial melting of rocks

located atop the slab and inside the plume. Crustal growth rates increase with time before reaching a steady state ( $60\text{km}^3/\text{km}/\text{Myrs}$ ). The mantle component of the newly produced crust decreases with time. Subduction in an extensional arc setting results in decompression melting of dry peridotite. The backward motion of the subduction zone relative to the motion of the plate leads to thinning of the overriding plate. Thus, hot and dry asthenosphere rises into the neck as the slab retreats, triggering decompression melting of dry peridotite. As a result crustal growth rates increase to values of about  $100\text{km}^3/\text{km}/\text{Myrs}$ .

### **An advanced mechanism driving Rift Induced Delamination: Melt Induced Weakening**

Melt induced weakening (MIW) is studied as a driving mechanism for rift induced delamination. Under MIW we conceive the mechanism of incipient melt generation in the upper asthenosphere by additional heating. Percolation and accumulation of this partial melt lump to regions with high melt fractions, where above a certain threshold melt is extracted and transferred to a higher level, assuming short time scale transport mechanisms such as dyking or channeling. Repeated emplacement within the mantle lithosphere or even in the lower crust causes the melt's heat to weaken its vicinity and so advective heat transport is accelerated. Petrological and geochemical arguments (Foley et al., 2009) enforce this view.

Rift induced delamination (RID) has been proposed as a geodynamic process explaining the extreme elevation of the Rwenzori Mountains in the western branch of the East African Rift System (Wallner and Schmeling, 2010). The special situation of two approaching rift tips with a finite offset is given by the southward propagating Albert Rift and the northward spreading Edward Rift encircling almost completely the old metamorphic horst. We assume, upwelling asthenosphere below the rifts surrounds the stiff lithosphere; if there the viscosity and strength, especially in the lower crust, is sufficiently reduced, the delamination of cold and dense mantle lithosphere root may be triggered. As a consequence the less dense crustal block is unloaded and uplift is induced along steep inclining faults. Seismological observations, particularly seismicity distribution, low velocity zones seen in receiver functions (Wölbern et al., 2010) as well as in tomography and the location of an anomalously deep earthquake cluster (Lindenfeld et al., 2010) strengthen RID hypothesis.

Numerical modeling is based on thermo-mechanical physics of visco-plastic flow approximated by Finite Difference Method in an Eulerian formulation in 2D. The equations of conservation of mass, momentum and energy are solved for a multi component and two phase system. Temperature-, pressure- and stress-dependent rheology, based on laboratory data of appropriate samples are assumed for upper crust, lower crust and mantle.

Successful models applied a strong initial temperature anomaly within the lithosphere, driving the process. To replace this non-geological ad hoc initial condition, we test the more self-consistent MIW process. It is a positive feedback-system and may lead, if strong enough, to detachment of cold mantle lithosphere. Studies on parameter variations of the initial temperature perturbation reveal a restricted range for functioning RID models. The coincidence with the settings of the Rwenzori situation establishes the RID concept furthermore. Until now the Rwenzoris are the sole case where RID is applicable. To what extent this experience can be transferred to MIW is analyzed.

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# Mantle convection and the generation of geochemical reservoirs in the silicate shell of the Earth

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We present a dynamic 3-D spherical-shell model of mantle convection and the evolution of the chemical reservoirs of the Earth's silicate shell. Chemical differentiation, convection, stirring and thermal evolution constitute an inseparable dynamic system. Our model is based on the solution of the balance equations of mass, momentum, energy, angular momentum, and four sums of the number of atoms of the pairs  $^{238}\text{U}$ - $^{206}\text{Pb}$ ,  $^{235}\text{U}$ - $^{207}\text{Pb}$ ,  $^{232}\text{Th}$ - $^{208}\text{Pb}$ , and  $^{40}\text{K}$ - $^{40}\text{Ar}$ . Similar to the present model, the continental crust of the real Earth was not produced entirely at the start of the evolution but developed episodically in batches [1-7]. The details of the continental distribution of the model are largely stochastic, but the spectral properties are quite similar to the present real Earth. The calculated Figures reveal that the modelled present-day mantle has no chemical stratification but we find a marble-cake structure. If we compare the observational results of the present-day proportion of depleted MORB mantle with the model then we find a similar order of magnitude. The MORB source dominates under the lithosphere. In our model, there are nowhere pure unblended reservoirs in the mantle. It is, however, remarkable that, in spite of 4500 Ma of solid-state mantle convection, certain strong concentrations of distributed chemical reservoirs continue to persist in certain volumes, although without sharp abundance boundaries. We deal with the question of predictable and stochastic portions of the phenomena. Although the convective flow patterns and the chemical differentiation of oceanic plateaus are coupled, the evolution of time-dependent Rayleigh number,  $Rat$ , is relatively well predictable and the stochastic parts of the  $Rat(t)$ -curves are small. Regarding the juvenile growth rates of the total mass of the continents, predictions are possible only in the first epoch of the evolution. Later on, the distribution of the continental-growth episodes is increasingly stochastic. Independently of the varying individual runs, our model shows that the total mass of the present-day continents is not generated in a single process at the beginning of the thermal evolution of the Earth but in episodically distributed processes in the course of geological time. This is in accord with observation. Finally, we present results regarding the numerical method, implementation, scalability and performance.

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## **Equatorially anti-symmetric convection in rotating spherical shells**

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The ancient dynamos in Earth and Mars likely operated without an inner core being present and were thus exclusively driven by secular cooling. Numerical simulations show that the related convective motions are particularly sensitive to the outer boundary condition. The lower mantle pattern as well as larger impact events may thus have had a profound influence on the core dynamics and the magnetic field pattern in the early dynamos of Earth and Mars.

We use numerical simulations to explore the impact of a boundary induced equatorial symmetry breaking on non-magnetic convection. The secular cooling is modeled by homogeneous volumetric heat sources, a flux boundary condition is used at the outer boundary, and the flux is set to zero at the boundary of the inner core which is retained for numerical reasons. While bottom heated spherical shell convection is typically dominated by equatorially symmetric motions this changes for volumetric heating. When the Rayleigh number exceeds a critical value, equatorially asymmetric convection modes set in, even for homogeneous boundary conditions. These modes are much more easily excited when the equatorial symmetry is broken via the outer boundary flux condition. Small flux variations suffice to yield a flow that is clearly dominated by equatorially asymmetric thermal winds. The effect, however, decreases with the Ekman number due to the more severely enforced Taylor-Proudman theorem. We work on scaling our results to the planetary situation.